Second Annual Herzog Prize
Examination
November 9, 1974

Problem 1:   (Dr. T. Butts)

Suppose $p(x)$ is a polynomial with integral coefficients. If $p(x) = 150$ for 5 distinct integral values of x , then $p(x) \neq 439$ for all integral values of x .

Problem 2:   (Prof. J. Plotkin)

Let X be a set of n objects. Two subsets S and T of X are incomprable if neither $S \subset T$ nor $T \subset S$. What is the maximum possible number of subsets that can be pairwise incomprable?

Problem 3:   (Dr. T. Butts)

For which n does there exist in the plane n rational points (both coordinates rational) that are the vertices of a regular n-gon?

Problem 4:   (Prof. D. Wright)

You are a merchant in possession of 3000 bananas and one slothful gluttonous camel. You wish to transport your stock of bananas by camel for sale in a town 1000 miles distant. However your camel will carry no more than 1000 bananas at a time and will eat one banana for every mile he travels. What is the maximum number of bananas that you can deliver to your destination? What if you had 3001 bananas?

Problem 5:   (Prof. John Kinney)

Suppose $f(x)$ is a continuous real valued non-decreasing function on $[0,1]$ with $f(0) = 0$ and $f(1) = 1$.

a) Find the best possible upper and lower bound for the arc length of the curve $y = f(x)$ over $[0,1]$.

b) Are these bounds ever achieved?

Problem 6:   (Dr. T. Butts)

Let $r_1 , \ldots , r_n$ be the roots of the polynomial

$$p(x) = 1 + x + x^2 + \cdots + x^n$$

and let $s_1 , \ldots , s_{n-1}$ be the roots of $p'(x)$. Evaluate

$$\sum_{i,j} \frac{1}{r_i - s_j}$$