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MATH 235, Spring, 2009, – EXAM #3 – Newhouse
7 pages—including tables total = 80 points

NO CALCULATORS, SHOW ALL WORK

1. For each of the following functions $f(t)$, find the Laplace transform $F(s)$.

   (a) (10 points)
   $f(t) = 4cos(2t) + sin(t)$ $F(s) =$

   (b) (15 points)
   $f(t) = \begin{cases} 1 - t^2, & 0 \leq t < 3 \\ t^2, & t \geq 3 \end{cases}$ $F(s) =$

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2. For each of the following functions $F(s)$ find the inverse Laplace transform $f(t)$

   (a) (10 points)
   $F(s) = \dfrac{e^{-2s}}{s^2 - 4s + 5}$ $\qquad f(t) =$

   (b) (10 points)
   $F(s) = \dfrac{s+1}{s^2 - 8s + 8}$ $\qquad f(t) =$

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3. (10 points)
   Find the general solution to the equation $x^2y'' - 2xy' + 2y = 0$

4. (15 points)
   Solve the following initial value problem using Laplace transforms.

$$y'' + 4y' + 4y = 2e^{-t},\ y(0) = 1,\ y'(0) = 1$$

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5. Find the general solution to each of the following systems and sketch the solution curves.

   (a) (15 points)

$$\begin{aligned} x' &= x - 4y \\ y' &= 4x + y \end{aligned}$$

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(b) (15 points)

$$\begin{aligned} x' &= 4x - y \\ y' &= 6x - y \end{aligned}$$

A short table of integrals

(a) A a constant $\implies \int A dx = Ax + C$,

(b) $n \neq 0, 1 \implies \int x^n dx = \frac{x^{n+1}}{n+1} + C$

(c) $\int \frac{dx}{x} = log(x) + C$; Integration by parts: $\int u dv = uv - \int v du$

(d) $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$; Substitution: $u = u(x)$, $\int u du = \int u(x) u'(x) dx$

(e) $\int x e^{ax} dx = e^{ax}(\frac{x}{a} - \frac{1}{a^2}) + C$

(f) $\int x^2 e^{ax} dx = e^{ax}(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}) + C$

(g) $\int log(ax) = xlog(ax) - x + C$

(h) $\int xlog(ax) = \frac{x^2}{2} log(ax) - \frac{x^2}{4} + C$

(i) $\int \frac{dx}{a^2+x^2} = \frac{1}{a} arctan(\frac{x}{a}) + C$

(j) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} log(\frac{x+a}{x-a}) + C$

(k) $\int \frac{dx}{\sqrt{a^2-x^2}} = arcsin(\frac{x}{a}) + C$

(l) $\int \frac{dx}{\sqrt{x^2-a^2}} = log(x + \sqrt{x^2 - a^2}) + C$

(m) $\int \frac{dx}{\sqrt{a^2+x^2}} = arcsinh(\frac{x}{a}) + C$

(n) $\int sin(ax) = -\frac{1}{a} cos(ax) + C$

(o) $\int cos(ax) = \frac{1}{a} sin(ax) + C$

(p) $\int tan(ax) = \frac{1}{a} log(sec(ax)) + C$

(q) $\int cot(ax) = \frac{1}{a} log(sin(ax)) + C$

(r) $\int sec(ax) = \frac{1}{a} log(tan(ax) + sec(ax)) + C$

(s) $\int csc(ax) = -\frac{1}{a} log(csc(a * x) + cot(a * x)) + C$

(t) $\int x\ cos(ax) = \frac{x\ sin(ax)}{a} + \frac{cos(ax)}{a^2} + C$

(u) $\int x\ sin(ax) = \frac{sin(ax)}{a^2} - \frac{x\ cos(ax)}{a} + C$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 7
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28