

7. A bacteria culture starts with 840 bacteria and grows at a rate proportional to its size. After 2 hours there will be 1680 bacteria.

∴ (1) As the bacteria grows at a rate proportional to its size, so the differential equation should be.

$$\frac{dP}{dt} = rP(t) \quad P_0 = 840 \quad P_0 \text{ is the initial value.}$$

solve this DE, we get.

$$840 \cdot e^{\frac{(\ln 2)t}{2}} \quad \text{here } t \text{ means } t \text{ hours.}$$

(2) What will be the population after 6 hours?

$$P(6) = 840 \cdot e^{\frac{\ln 2}{2} \cdot 6}$$

$$= 840 \cdot e^{3 \ln 2}$$

(3) How long it will take to reach 2170?

$$P = 840 \cdot e^{\frac{\ln 2}{2} \cdot t}$$

as we know $P = 2170$, just solve for t .

$$2170 = 840 \cdot e^{\frac{\ln 2}{2} \cdot t}$$

$$\frac{2170}{840} = e^{\frac{\ln 2}{2} \cdot t}$$

$$\ln\left(\frac{2170}{840}\right) = \frac{\ln 2}{2} \cdot t$$

$$t = \ln\left(\frac{2170}{840}\right) / \frac{\ln 2}{2}$$

$$= 2 \ln\left(\frac{2170}{840}\right) / \ln 2$$

8. A cell of some bacteria divides into two cells every 40 minutes.

The initial population is 3 bacteria.

(a) Find the size of population after t hours.

~~8.~~ ~~8.~~ $40 \text{ min} = \frac{2}{3} \text{ hour}$.

It means every $\frac{2}{3}$ hour, the size of population will become 2 times as before.

so the relationship will be

$$y(t) = y_0 \cdot 2^{\frac{t}{2/3}} = y_0 \cdot 2^{\frac{3}{2}t}$$

$$y_0 = 3. \text{ so}$$

$$y(t) = 3 \cdot 2^{\frac{3}{2}t}$$

(b) Find the size of population after 7 hours.

~~8.~~ $y(7) = 3 \cdot 2^{\frac{3}{2} \cdot 7}$ #

(c) When will the population reach 21?

~~8.~~ we know y , and then solve for t .

$$21 = 3 \cdot 2^{\frac{3}{2}t}$$

$$7 = 2^{\frac{3}{2}t}$$

$$\log_2 7 = \frac{3}{2}t$$

$$t = \frac{2}{3} \cdot \log_2 7$$

9. A thermometer is taken from a room where the temperature is 24°C . to the outdoors, where the temperature is -4°C . After 1 minute the thermometer reads 14°C . (a) the reading after 5 more minutes?

P. ~~10~~ Using Newton's Law of Cooling:

$$\Theta = T + (\Theta_0 - T)e^{kt} \quad (*)$$

From the context we know.

$$\Theta_0 = 24 \quad T = -4.$$

so (*) becomes:

$$\Theta = -4 + (24 - (-4))e^{kt}$$

$$\Theta = -4 + 28e^{kt}$$

also we know after 1 min, it's ~~temp~~ reads 14°C

so

$$\Theta(1) = 14 = -4 + 28e^{k \cdot 1}$$

$$\frac{18}{28} = e^k$$

$$k = \ln \frac{18}{28}$$

so the function is:

$$\Theta(t) = -4 + 28e^{t \ln \frac{18}{28}}$$

After 5 more minutes, it will be 6 minutes, so.

$$\Theta(6) = -4 + 28e^{6 \ln \frac{18}{28}}$$

(b). When will the thermometer read -3°C ?

$$\textcircled{b).} \quad \Theta(t) = -3 = -4 + 28e^{t \ln \frac{18}{28}}$$

solve it for t , we get.

$$1 = 28e^{t \ln \frac{18}{28}}$$

$$\frac{1}{28} = e^{t \ln \frac{18}{28}}$$

$$\ln\left(\frac{1}{28}\right) = t \cdot \ln \frac{18}{28}$$

$$t = \left(\ln \frac{1}{28}\right) / \left(\ln \frac{18}{28}\right)$$