Math 950, Spring 2009

Course: Numerical Methods for Partial Differential Equations I

Time & Location: T/Th, 2:40-4:00pm, C308 WH

Professor: Di Liu

Email: richardl@math.msu.edu

Office: A201 Wells Hall

Office hours: T/Th: 4:30pm-5:30pm, Th: 11:00am-12:00pm, and by appointment.

Class webpage: Homework assignments and supplemental materials will be given here.
http://www.math.msu.edu/∼richardl/teaching/math950

Course Description: This graduate level course is about the numerical solutions of Partial Differential Equations using Finite Difference methods. We will introduce techniques for analyzing the convergence of the schemes and evaluate their advantages and limitations. Emphasis will be also on the implementations.

Prerequisites: Programming skills with Fortran, C/C++, or Matlab are necessary. The students are expected to have taken ODE/PDE courses at the advanced undergraduate level. Previous knowledge on Numerical Analysis is also required.

Textbooks:

- *Finite difference methods for ordinary and partial differential equations: steady-state and time-dependent problems*, R. LeVeque, 2007.

  This will be our primary reference for the course. Exercises for each chapter are available at the website of the book, where you can also find Matlab codes for some of the algorithms.

  http://www.amath.washington.edu/∼rjl/fdmbook/index.html

- *Finite difference schemes and partial differential equations*, J. Strikwerda, 2nd ed., 2004.
- *Numerical solutions of partial differential equations*, K. W. Morton and D. F. Mayers, 2nd ed., 2005.
- *A first course in the numerical analysis of differential equations*, A. Iserles, 1996.
- *Spectral methods in Matlab*, L. N. Trefethen, 2000.
- *A multigrid tutorial*, W. L. Briggs, V. E. Henson and S. F. McCormick, 2000.

Grading policy: 60% on homeworks; 40% on final project.

Homework: Home works will be assigned every other week from the second week. A due date will be denoted on each assignment and no late homework will be accepted except extreme situations. There will be approximately 6 homework assignments in total. Each homework set will consist of a few problems regarding proofs and a couple of other problems for code implementation. A sample homework can be found on the website of the LeVeque book. When collected,

- the homework must be prepared with Latex. Latex is a must skill for mathematicians. Here is a helpful link. http://www-h.eng.cam.ac.uk/help/tpl/textprocessing
- the numerical results must be illustrated with Tables and Figures in the format indicated in the problems. You can generate figures using Matlab or Gnuplot. A tutorial for Matlab can be found at http://www.math.mtu.edu/%7Emsgocken/intro/intro.html

  A free GNU software called Octave provides computational environment compatible with matlab. Details can be found at http://www.gnu.org/software/octave/
- the code must submitted together with the homework. In Unix, the following command produces a readable print of a program in a two-column landscape format:
  enscript -G2r filename

Final Project: A final project will be given to make sure that things can be put together. Details will be announced later.

Schedule: We will try to follow the following schedule which is still subject to change.

Week 1,2: Introduction, Boundary Value Problems

1. Finite difference approximation
2. Consistency, stability and convergence
3. Neumann boundary condition
4. Linear and Non-linear equations
5. Singular perturbation, Non-uniform grid
6. Higher order and spectral methods

Week 3,4: Elliptic Equations

1. Analysis of elliptic equations
2. 5-point and 9-point Laplacian methods
3. Divergence form equations
4. Non-rectangle boundary
5. Coordinate change

Week 5,6: Iterative Methods for Sparse Linear Systems

1. Jacobi, Gaussian-Seidel, SOR
2. Conjugate Gradient
3. Arnoldi process and GMRES
4. Multigrid

Week 7,8,9: Parabolic Equations

1. Analysis of parabolic equations
2. MOL (Explicit, Implicit, Crank-Nicolson)
3. Stiffness, stability and convergence

4. Von-Neumann analysis, Maximum principle
5. Neumann boundary conditions
6. Variable coefficients and non-linear problems
7. ADI, LOD for higher dimensions
8. Polar coordinate and curved boundaries

Week 10,11,12: Hyperbolic Equations

1. Analysis of hyperbolic equations
2. MOL (Lax-Wendroff, Upwind, ...)
3. Von-Neumann analysis, CFL condition
4. Conservation Laws, Finite Volume methods
5. Dissipation and dispersion
6. Hamiltonian systems
7. Modified equation analysis
8. Equations in higher dimensions
9. Initial boundary value problems

Week 13,14,15: Mixed Equations, Selected topics

1. Second order equations
2. MOL, Taylor series methods
3. Time stepping
4. Eno&Weno methods

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