Math 950 Final project Due 05/08/09

Prob 1 (10pt). Use an explicit scheme to approximate the Poisson equation:

$$u_{xx} + u_{yy} + xe^{-x} = 0, \tag{1}$$

in the region defined by $x \geq 0$, $y \geq 0$, $x^2 + y \leq 1$, with a uniform grid size $\frac{1}{3}$. The boundary conditions are $u(x, 0) = x$, $u_x(0, y) = 0$ and $u(x, 1 - x^2) = 1$.

Prob 2 (10pt). Use the ADI and LOD methods to solve the Heat equation:

$$u_t = u_{xx} + u_{yy}, \tag{2}$$

in the square region $0 \leq x \leq 1$, $0 \leq y \leq 1$. The boundary conditions are $u_x = 0$ on the side $x = 0$ of the square and $u = 2 - x^2 - y^2$ on the rest of the boundary. The initial condition is set to be $u = 0$.

Prob 3 (20pt). The linearized one-dimensional forms of the isentropic compressible fluid equations are

$$\begin{aligned} \rho_t + q\rho_x + \omega_x &= 0, \\ \omega_t + q\omega_x + a^2\rho_x &= 0, \end{aligned} \tag{3}$$

where a and q are positive constants.

(a) Show that an explicit scheme which uses central differences for the x-derivatives is always unstable.

(b) By adding the extra terms arising from a Lax-Wnedroff scheme, derive a conditionally stable scheme and find the stability condition.