Math132 Midterm 4 12/05/09

Show intermediate steps!

1.) Evaluating the following integrals

(a) (5pt)

$$\int \left(3\sin x - \frac{3}{x^2}\right) dx$$

$$= -3\cos x - 3\frac{x^{-2+1}}{-2+1} + C$$

$$= -3\cos x + \frac{3}{x} + C$$

(b) (5pt)

$$\int_0^9 \frac{t-\sqrt{t}}{t^2} dt$$

(c) (5pt)

$$\int_0^1 (x^2-1)\sqrt[3]{\frac{x^3}{3}-x}\,dx$$

$u = x^3/3 - x$

$\frac{du}{dx} = x^2 - 1$

$$= \int_{0^3/3-0}^{1^3/3-1} u^{1/3}\,du$$

$$= \int_0^{-2/3} u^{1/3}\,du$$

$$= \left.\frac{u^{1/3+1}}{1/3+1}\right|_0^{-2/3}$$

$$= \frac{3}{4}\left(-2/3\right)^{4/3}$$

$$= 2\sqrt[3]{2/3}$$

(d) (5pt)

$$\int x^2\cos(x^3)\sin^2(x^3)dx$$

$u = x^3$

$$= \frac{1}{3}\int \cos u \sin^2 u\,du$$

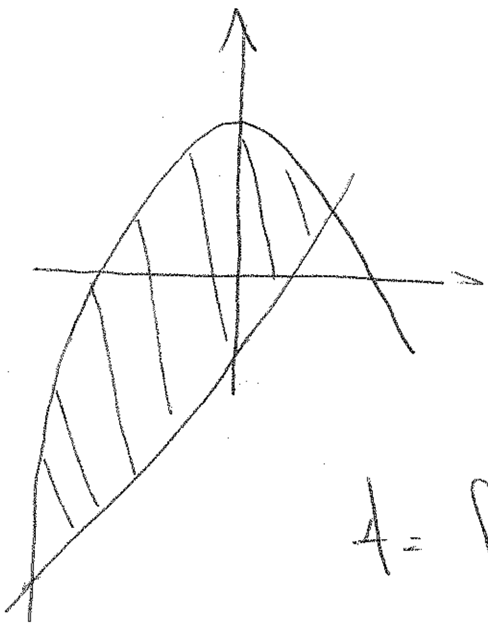
$v = \sin u$

$$= \frac{1}{3}\int v^2\,dv$$

$$= \frac{1}{9}v^3 + C$$

$$= \frac{1}{9}\sin^3(x^3) + C$$

2.) (10pt) Find the area of the region bounded by the curve $y = 2 - x^2$ and $y = 2x - 1$.



$$\begin{cases} y = 2 - x^2 \\ y = 2x - 1 \end{cases}$$

$$\Rightarrow \quad 2 - x^2 = 2x - 1$$

$$\Rightarrow \quad x^2 + 2x - 3 = 0$$

$$\Rightarrow \quad x = -3, 1$$

$$A = \int_{-3}^{1} \left((2 - x^2) - (2x - 1) \right) dx$$

$$= \int_{-3}^{1} (3 - 2x - x^2) \, dx$$

$$= 3 \cdot x - x^2 - x^3/3 \Big|_{-3}^{1}$$

$$= (3 \cdot 1 - 1^2 - 1^3/3) - (3 \cdot (-3) - (-3)^2 - (-3)^3/3)$$

$$= 10\tfrac{2}{3}$$

3.) (10pt) Find the derivative of the following function

$$f(x) = \int_0^{\sqrt{x}} \tan(t^4 + 1) dt.$$

$u = \sqrt{x}$ $\qquad y = f(x) = \int_0^u \tan(t^4 + 1) dt$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \tan(u^4 + 1) \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \tan(x^2 + 1)$$