

On the stability of one-dimensional diffusion flames established between plane, parallel, porous walls

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Abstract

A one-dimensional, non-premixed flame stability analysis is undertaken. Oscillatory and cellular flame instabilities are identified by a careful study of the numerically calculated eigenvalues of the linearized system of equations. The numerical investigation details the critical locations for changes in flame behaviour, as well as the critical values of various parameters that affect flame stability. A critical Lewis number, greater than unity, is identified as the value where unstable oscillations may begin to appear ($Le > Le_c$) and for which cellular flames can exist ($Le < Le_c$). Some prior discussions are clarified regarding the aforementioned critical values, as well as the role of convection in producing flame instabilities. The methodology of the stability analysis is discussed in detail.

Nomenclature

A, B, C, E	Generic points along the 'S' curve
D	Damköhler number
E	Activation energy
K	Wavenumber
Le	Lewis number
Pe	Peclet number
R	Ideal gas constant
T	Non-dimensional temperature
t	Non-dimensional time
w	Non-dimensional reactivity = $Dy_O y_F \exp(-T_a/T)$
x, y	Non-dimensional spatial coordinates
Y	Species mass fraction
y	Non-dimensional species mass fraction
Z	Mixture fraction

Greek

β	Zeldovich number
ϕ	Global stoichiometric coefficient
ϕ	Generic scalar variable
λ	Wavelength
τ, δ, Γ	Arbitrary spatial function

Subscripts

a	Activation reference
c, crit	Critical value
F	Fuel
f	Flame
O	Oxidizer
o	Reference
max	Maximum
new	New

Superscripts

/	Perturbation term
—	Steady-state term

1. Introduction

In the study of diffusion flames the question of stability naturally arises when the words ‘ignition’ and ‘quenching’ are used. These are ‘near-limit’ phenomena in the sense that either the temperature or the reactant concentrations (or both) can barely support combustion, making the transition to either a burning state for the former and a non-burning state for the latter a distinct possibility. These concepts can be broadened when other physical, geometric and parametric influences are examined, such as nearby cold surfaces in the case of spreading flames over solid and liquid fuel, or multi-step chemistry, or disparate Lewis numbers of the reactants, or radiant losses from the gas and from soot particulates near the flame, for example. Before flame extinction concepts are broadened, however, the skeletal phenomenon should be understood in the clearest possible terms. The present paper attempts to create such an understanding for a physically idealized model of diffusion flame stability.

In addition, a second goal is sought, namely a clear exposition of diffusion flame stability for pedagogical purposes. The authors believe this is absent in the literature. Our interest in diffusion flame stability lies in applications to flame spread. We found, however, that there was no complete exposition of the stability calculation for even the simplest model problems. The original study of Kirkby and Schmitz [1] comes closest to a full description but is hampered by the use of a difficult terminology.

The mathematical analysis of the stability of partial differential equations has a long history. It has evolved over the last few decades into a theory based on the functional analysis of PDEs in terms of their eigenvalue spectra. The analysis of stability from a mathematical standpoint amounts to the examination of the eigenvalue spectrum of the linearized system of equations in relation to the original spectrum of the nonlinear system of equations. For infinitesimal disturbances the confluence of the two systems can be rigorously demonstrated, thereby rendering the analysis of the linearized problem mathematically representative of the original (nonlinear) problem.

The eigenvalue spectra can be examined in numerous ways. The study by Kim *et al* [2] has, for the case $Le < 1$, postulated simplified forms of the conservation equations on either side of the flame sheet and then used asymptotic methods to derive ‘jump’ conditions across the flame sheet with the goal of obtaining formulae for the dependence of the eigenvalues on the parameters D , Le and Pe . Extending this work with asymptotics, Kim [3] included higher-order terms in the expansion of the inner zone solution in a more thorough examination of the effects of non-unity Lewis number. The subsequent work of Sohn *et al* [4] employs no asymptotic methods in $Le > 1$ studies of diffusion flame stability. With regard to the latter work, which demonstrates a more complicated response³ than the $Le < 1$ case, stability is examined by integrating the conservation equations numerically. Transient evolution of a monotonically growing solution indicates instability.

The overall nature of the spectrum of the eigenvalues for the diffusion flame can be understood both in a ‘physical’ way and from experimental observations. Consequent to Linan’s analysis of diffusion flame structure [5], Peters [6] determined by physical arguments that the middle branch instability must be a ‘fast time’ instability. The ‘fast time’ stability discussion is the result of a simple scaling analysis: if the reaction sheet is thin, of order [activation energy]⁻¹, then in order to retain the transient term in the diffusion equation the temporal scale factor must be of order [activation energy]⁻¹ times smaller than the spatial scale. Laboratory observations have also produced a physical understanding of the nature and types of instabilities (cellular, oscillatory, etc). Among these experimental works are [7, 8], which deal with flame instability in many configurations. In addition, many spectacular photographs can be found of ceiling flame instabilities, see especially [9].

Our analysis will demonstrate that the eigenvalue spectrum is vitally important, especially for the case $Le > 1$ examined recently by Cheatham and Matalon [10] and Kim *et al* [4]. We shall demonstrate that the tracking of the leading eigenvalue is insufficient to understand the instability process. The behaviour of the second and third eigenvalues often signals imminent transitional behaviour.

2. Model

The proposed model is a one-dimensional version of the film diffusion flame examined by Kim *et al* [2]. There, the authors were interested in the formation of striped instability patterns in a one-dimensional flame sheet. A minimum of two spatial dimensions was required to describe the pattern of instability: one coordinate lay along the flame sheet while the other lay perpendicular to the flame sheet. The coordinate along the flame sheet described the striping pattern. The fragmented flame sheets have been called ‘flame tubes’ [11]. In this study, we are interested in the fundamental mathematical description of the instability mechanism. This does not require the formation of different sorts of flame patterns. For this reason, following Kirkby and Schmitz [1], we retain as little detail as required in order to produce a mathematical description of diffusion flame stability. The in-flame coordinate is thus deemed inessential for the stability analysis.

The proposed one-dimensional model consists of flowing fuel and diffusing oxidizer species on opposite sides of the reaction sheet. Radiative heat losses have been ignored, as have both Soret and Dufour diffusion. The flow is assumed to be uniform. The chemistry is modelled with a single-step, irreversible, Arrhenius reaction. Figure 1 shows a schematic of the one-dimensional model that has been used in previous stability analyses by Kim and

³ The case for $Le < 1$ is less complicated in that no upper branch instabilities are seen and the middle branch is unstable. Furthermore, no oscillations are present for this case as can be seen when $Le > 1$.

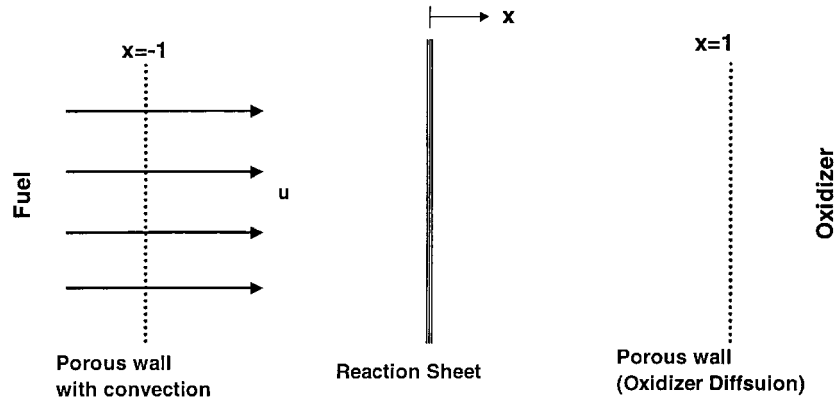


Figure 1. Schematic diagram of convective film diffusion flame.

colleagues [2, 4] and Matalon and co-workers [10, 12]. The latter investigators also describe a realistic physical configuration from which this model could, in principle, be realized in a laboratory experiment. The problem formulation proceeds along similar lines as that of [2], producing the following set of non-dimensional equations:

$$\frac{\partial T}{\partial t} + Pe \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} + w \quad (1a)$$

$$Le \frac{\partial y_i}{\partial t} + PeLe \frac{\partial y_i}{\partial x} = \frac{\partial^2 y_i}{\partial x^2} - w \quad i = O, F. \quad (1b)$$

The equations are coupled through the nonlinear reaction term, $w = Dy_O y_F \exp(-T_a/T)$. A single Lewis number is employed for both reactants. All terms in equations (1) are defined in the nomenclature.

The boundary conditions for equations (1) are:

$$\begin{aligned} T = T_0 \quad y_F = 1 \quad y_O = 0 \quad \text{at } x = -1 \\ T = T_0 \quad y_F = 0 \quad y_O = 1 \quad \text{at } x = +1. \end{aligned} \quad (2)$$

In this paper we assume, for simplicity, that the fuel and oxidizer inflow temperatures are both equal to a reference temperature, T_0 . The symmetry of equation (1) and the boundary conditions imply that y_O and y_F are interchangeable; hence, from a stability perspective this formulation describes both blowing fuel and blowing oxidizer.

The configuration of figure 1 is a generic, simplified version of many combustion problems. These include droplet combustion (blowing fuel from the droplet, inward oxygen diffusion from the ambient), spray flames, jet flames, flame spread and diffusion flames in channels, among others. More complicated configurations require additional geometry-dependent terms; and more complicated model formulations may require the inclusion of buoyancy or variable properties or pressure variations, or even magnetic fields, but the qualitative nature of the solutions generated herein shall be representative, as a model, for all of these configurations.

3. Steady-state solutions

3.1. Analytical Burke–Schumann solutions

The character of the steady-state solution provides insight into the influences of disturbances on the flame's ultimate stability or instability. For clarification of subsequent discussions, we

present the steady Burke–Schumann solutions and briefly describe their structure. The solution for the mixture fraction is

$$\begin{aligned} Z &= [\exp(LePe(1+x)) - 1] / [\exp(2LePe) - 1] \\ &\simeq \frac{(1+x)}{2} [1 - LePe(1-x)/2 + O(LePe)^2]. \end{aligned} \quad (3)$$

When $Pe = 0$ we have $Z = (1+x)/2$. It is relatively straightforward to show that when Pe is non-zero, Z is smaller everywhere (except at $x = \pm 1$) than the $Pe = 0$ Z -distribution, i.e. $Z_{Pe>0} < Z_{Pe=0}$. The flame is shifted to the right by the blowing fuel, and this shift increases as Pe increases. We will not examine the large Pe case in this paper.

At the flame sheet $Z = \frac{1}{2} = Z_f = [\exp(LePe(1+x_f)) - 1] / [\exp(2LePe) - 1]$, which can be solved to give $x_f = \ln[\cosh(LePe)]$ for the flame sheet location. Burke–Schumann solutions for oxidizer, fuel and temperature distributions are obtained by solving equations (1) with w replaced with a delta function.

Oxidizer side ($-1 < x < x_f$):

$$Y_O = 0 \quad (4a)$$

$$Y_F = 1 - \frac{\exp(LePe(x/2)) \sinh(LePe(1+x)/2)}{\exp(LePe(x_f/2)) \sinh(LePe(1+x_f)/2)} \quad (4b)$$

$$T - T_0 = (T_f - T_0) \frac{\exp(Pe(x/2)) \sinh(Pe(1+x)/2)}{\exp(Pe(x_f/2)) \sinh(Pe(1+x_f)/2)}. \quad (4c)$$

Fuel side ($x_f < x < 1$):

$$Y_O = 1 - \frac{\exp(LePe(x/2)) \sinh(LePe(1-x)/2)}{\exp(LePe(x_f/2)) \sinh(LePe(1-x_f)/2)} \quad (4d)$$

$$Y_F = 0 \quad (4e)$$

$$T - T_0 = (T_f - T_0) \frac{\exp(Pe(x/2)) \sinh(Pe(1-x)/2)}{\exp(Pe(x_f/2)) \sinh(Pe(1-x_f)/2)}. \quad (4f)$$

These distributions are applicable in the regions on either side away from the flame location. From these distributions it is possible to deduce some important facts (see [2]). First, the outer-zone formula for T_f produces a Le dependence in which T_f increases as Le decreases and T_f decreases as Le increases. The roughly Le^{-1} dependence of T_f is well known and has been exploited in more detailed formulations of the non-dimensional combustion equations [2]. Second, the general shape of the profiles in response to the flow is ascertained, as is the response of the flame position to the fuel inflow.

3.2. Numerical solutions

The steady-state equations (1a) and (1b) can be solved numerically by standard methods. The numerical solution produces the characteristic ‘S’ curve which plots T_{\max} versus D , where T_{\max} is the maximum temperature in the domain. This ‘S’ curve, to the knowledge of the authors, was first produced and thoroughly examined by Fendell [13]. The top branch of the curve corresponds to steady burning (under most *but not all* conditions), the middle branch is unstable, and the lower branch is known as the ignition branch (see figure 2). The real ‘S’ curve does not look like an actual S since the lower turn occurs at a value of D that is usually many orders of magnitude larger than its value at the upper turn.

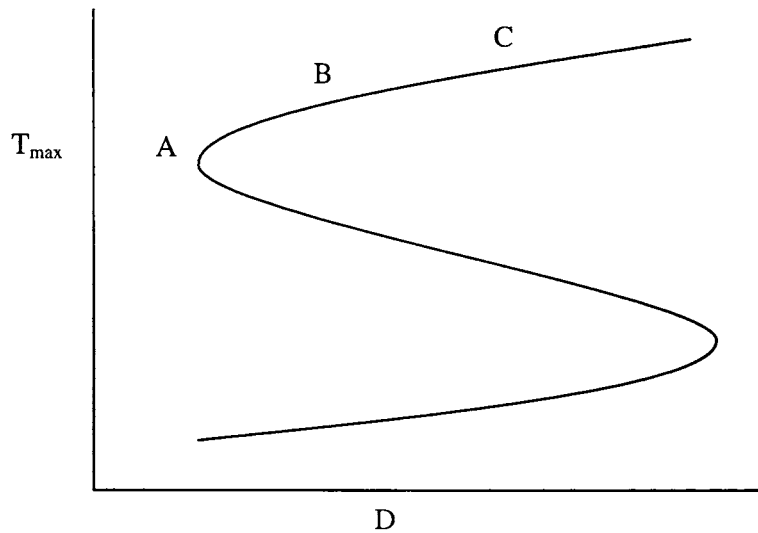


Figure 2. Qualitative plot of the one-dimensional non-premixed flame temperature versus the Damköhler number. Three distinct branches are traditionally represented. The upper branch (above A) represents steady burning, the middle branch is unstable and the lower branch corresponds to the ignition branch. In actual plots the lower turn appears at values of D many orders of magnitude higher than D at the upper turn.

At the lower turning point the Arrhenius exponential is $O(\exp(-E/RT_0))$, whereas at the upper turning point the Arrhenius exponential is $O(\exp(-E/RT_f))$. The 'separation' in D between these two turning points is thus $O(\exp(E/R)(1/T_0 - 1/T_f))$, at least in the lowest approximation (i.e. without accounting for differences in reactant mass fractions at the two turning points).

An analysis of the 'S' curve using large activation energy asymptotics was carried out by Linan [5]. Combustion regimes in the various parts of the 'S' curve were elucidated and canonical lowest-order equations describing the mathematical character of each region were produced with an accuracy that improved as the 'Zeldovich number', β , increased. The expansions, which proceed for both inner (flame zone) and outer (chemical equilibrium zones) in inverse powers of β , can be continued, at least in principle, indefinitely. Inner and outer solutions are matched in the characteristic fashion outlined in [14]. The expansions in β^{-1} are valid everywhere along the 'S' curve but the relative accuracy of the solutions diminishes near the turning point where additional (higher-order) series terms are needed to maintain accuracy. The leakage of reactants was shown to be a cause of the transition from vigorous to non-vigorous burning. Numerous features of this S curve have been clarified since the work of Linan [5], but the fundamental distinctions made therein (and in the original work of Fendell [13]) remain valid. Though some aspects of the preceding description, such as the use of one-step chemistry, or the predominant leakage of the 'wrong' reactant, etc [15] are unsuitable for a more comprehensive approach, the appeal of the 'S' curve remains strong. Discussions of diffusion flame stability are referred almost exclusively to 'S'-curve solutions [2, 4, 10]. Extending or broadening them to more realistic flame processes, geometries and parametric ranges is desirable.

4. Stability analysis

Equations (1a) and (1b) form a set of coupled semi-linear parabolic equations [16, 17]. Hence the evolution of small disturbances is governed by linearized forms of these equations. These linearized equations are derived by writing the solution of each dependent variable as a combination of a steady solution and a perturbation:

$$\phi(x, t) = \bar{\phi}(x) + \phi'(x, t) \quad (5)$$

where ϕ can be T , y_O or y_F . Substituting equation (5) into equation (1) and retaining only the terms that are linear in ϕ' in the Taylor series expansion of w yields the following linear system:

$$\frac{\partial T'}{\partial t} + Pe \frac{\partial T'}{\partial x} = \frac{\partial^2 T'}{\partial x^2} + w' \quad (6a)$$

$$Le \frac{\partial y'_i}{\partial t} + PeLe \frac{\partial y'_i}{\partial x} = \frac{\partial^2 y'_i}{\partial x^2} - w' \quad i = O, F \quad (6b)$$

$$w' = D \left[\bar{y}_O \bar{y}_F \left(\frac{T_a}{\bar{T}^2} T' \right) + \bar{y}_O y'_F + y'_O \bar{y}_F \right] \exp \left(-\frac{T_a}{\bar{T}} \right) \quad (6c)$$

which is solved subject to the following boundary conditions (no disturbances at the boundaries):

$$T'(\pm 1, t) = y'_O(\pm 1, t) = y'_F(\pm 1, t) = 0. \quad (6d)$$

Since the spatial part is a linear operator with compact resolvent [17], the evolution of small disturbances is determined by the eigenvalues of equations (6a)–(6d). The eigenvalues are the complex numbers, σ , for which equations (6) possess a non-trivial solution in the form

$$T'(x, t) = \tau(x) \exp(\sigma t) \quad (7a)$$

$$y'_O(x, t) = \delta(x) \exp(\sigma t) \quad (7b)$$

$$y'_F(x, t) = \Gamma(x) \exp(\sigma t). \quad (7c)$$

If all eigenvalues have negative real parts, then all solutions of the nonlinear equations (1) that originate as small disturbances of the steady-state solution will decay exponentially to the steady solution [16, 17]. In this case the steady-state solution is said to be stable. On the other hand, if any of the eigenvalues has a positive real part then the steady-state solution is unstable, meaning that there exists a threshold disturbance size such that some arbitrarily small initial disturbances of the steady-state solution will evolve according to the nonlinear equations (1a) and (1b) to grow eventually past the threshold [16, 17].

With fixed parameters the steady-state solution was found first, using Mathematica's NDSolve in the shooting algorithm for the boundary value problem. Error controls in the software enable the successive elimination of subsequent higher-order error terms; the steady solution is for all practical purposes exact. In other words, the errors can be made as small as desired because it is known that the computed errors depend quadratically on grid size. Three different meshes were used (80, 160 and 320 grid points) to reduce the computational errors.

This steady solution was used in equations (6a)–(6d) along with equations (7a)–(7c) for the disturbance quantities. The equations were then discretized using a second-order central difference scheme. The resulting matrix eigenvalue problem was also solved using Mathematica. The computational errors of the eigenvalue also depend quadratically on the mesh size, hence we were able to reduce eigenvalue errors below a specified tolerance.

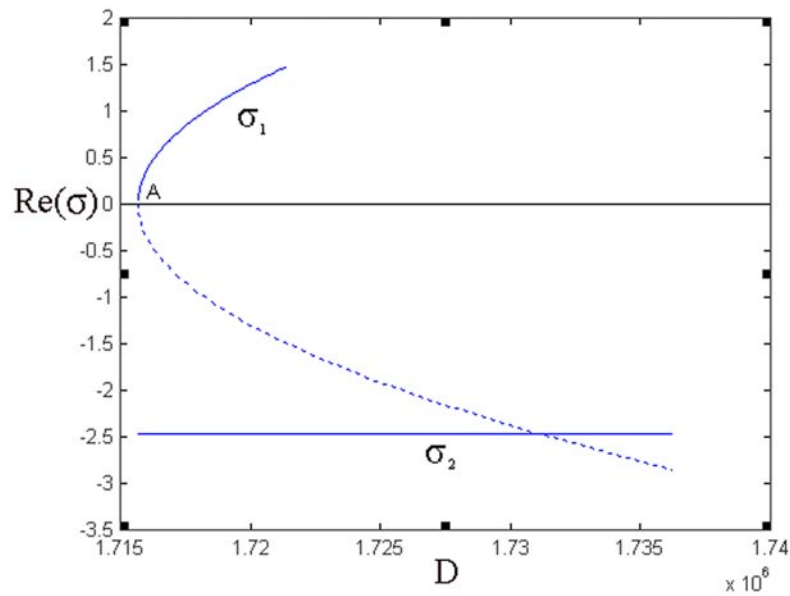


Figure 3. Leading two eigenvalues for the case of $Pe = 0$, $Le = 1$, $T_a = 4.0$, $T_0 = 0.05$. Here σ_2 has multiplicity 2 and is independent of D . Note that σ_1 is positive on the middle branch and negative on the upper branch (dotted curve).

For example, when $Pe = 0.5$, $Le = 0.9$, $T_0 = 0.05$, $T_a = 5$ and $D = 21\,190\,000$ we found that the leading eigenvalue on the middle branch was equal to 0.1612, and on the upper branch it was equal to -0.1522 . Many spot checks were made by using different numerical methods. In particular, a Fortran-based finite-difference scheme was used (with no error controls) with 300 nodes to produce the steady solution, and the eigenvalues were calculated using standard Lapack subroutines.

The stability in the neighbourhood of the upper turn, point A in figure 2, on the ‘S’ curve has been investigated for a large set of parameters:

$$0 \leq Pe \leq 1 \quad 0.1 \leq Le \leq 4 \quad 1.5 \leq T_a \leq 6.5 \quad 0.02 \leq T_0 \leq 0.05.$$

A brief description of the findings appears below. In later sections lengthier discussions of their physical significance are presented. For larger values of Pe the eigenvalue behaviour is more complicated, however, and more detailed analysis is needed before concrete generalizations are made.

Identifying the Le effect on the eigenvalue behaviour is essential in understanding the various regimes of flame behaviour. When $Le \leq 1$ the leading eigenvalue is negative on the upper branch, implying stability, and positive along the middle branch, implying instability. The leading eigenvalue is equal to zero at the turn, point A (see figures 3 and 6). When $Le = 1$ a linear combination of equations (1), commonly called the Schvab–Zeldovich procedure, produces two stable linear, homogeneous heat equations with a convection term, and a nonlinear equation. The two heat equations are responsible for an eigenvalue of multiplicity 2 that does not change with D (see figure 3 curve denoted by ‘ σ_2 ’). This constant eigenvalue is given by

$$-(\pi/2)^2/Le - (Pe/2)^2Le. \quad (8)$$

This result is obtained for $y_O - y_F$ derived from equation (6b) which is a linear heat equation with a convective term, and can be solved explicitly. When $Le = 1$ the solution has multiplicity 2

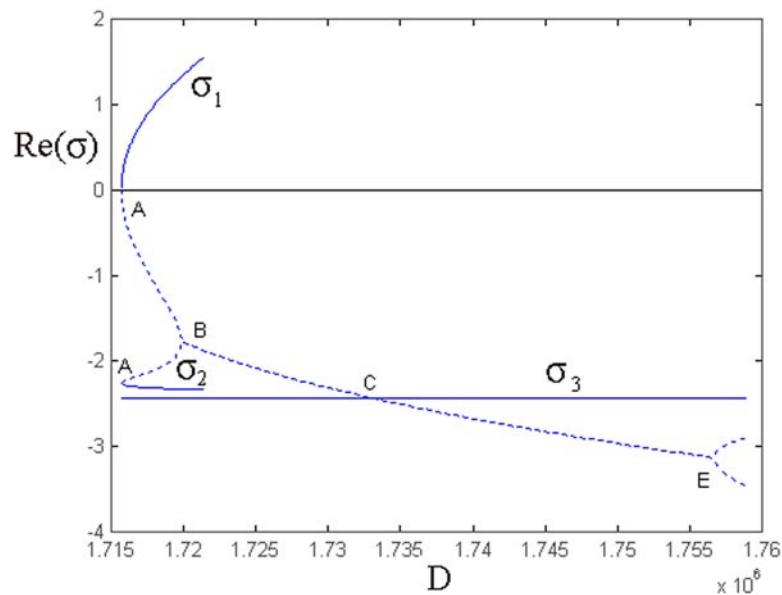


Figure 4. $1 < Le < Le_c$. Real eigenvalues σ_1 and σ_2 merge at B to form a complex conjugate pair until they split again at E. B moves up and E moves to the right as Le is increased. Here, $Pe = 0.0$, $Le = 1.01$, $T_a = 4.0$, $T_0 = 0.05$.

because $2T + y_O + y_F$ satisfies the same equation. The remaining nonlinear equation produces the eigenvalue denoted by ' σ_1 ' in figure 3. This eigenvalue is always positive on the middle branch, indicating instability as stated above. (Note that in all the figures pertaining to eigenvalue distributions, the dotted line indicates a position along the *upper* branch.) It is the character of the remaining two eigenvalues on this middle branch, however, that changes as Le is varied: the nature of this change dictates the stability of points along the sector ABC of the 'S' curve of figure 2.

When Le is slightly larger than unity, the eigenvalue that has multiplicity 2 when $Le = 1$ splits into two eigenvalues. One, σ_3 , is still given by equation (8) and the other one, σ_2 , interacts with σ_1 as shown in figure 4. σ_1 and σ_2 are negative real numbers between the turning points A and B on the upper branch (see figure 2). At point B they merge to form a complex conjugate pair with negative real parts (indicating decaying oscillations) until point E where they again split. At point C, σ_3 becomes the leading eigenvalue. As Le approaches unity the region between B and E shrinks to point C where σ_1 meets σ_2 in figure 3. As Le increases the value of σ_1 at B increases, B approaches A, and B reaches A at some critical value $Le_c > 1$.

When Le increases past the critical value Le_c , point B moves into the region of positive eigenvalues, $Re(\sigma) > 0$, and the characteristic behaviour in region ABC of figure 2 is changed completely (see figure 5). Note that figure 5 ($Le > Le_c$) is qualitatively the same as figure 4 ($Le < Le_c$) with the point E (not shown) moving rapidly towards higher D as Le is increased. When $Le > Le_c$ the region between points A and B on the upper branch has eigenvalues that are real and positive, indicating that perturbation growth will occur and lead to flame extinction. Between B and C the two leading eigenvalues form a complex conjugate pair, whose real part is positive at B and decreases as D increases. The real parts become zero at a point C' between B and C indicating damped oscillations until point C (not shown) where the *leading* eigenvalue is independent of D .

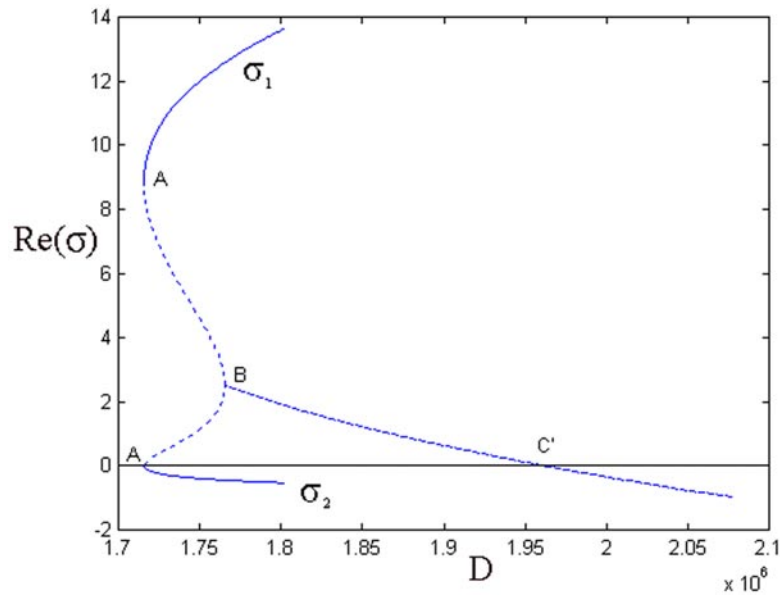


Figure 5. Leading two eigenvalues for $Le = 2.0$, $T_a = 4.0$, $Pe = 0$, $T_0 = 0.05$. Upper branch instabilities are seen up to point C' . Oscillations are unstable between B and C' and stable beyond C' . Pure perturbation growth exists between A and B .

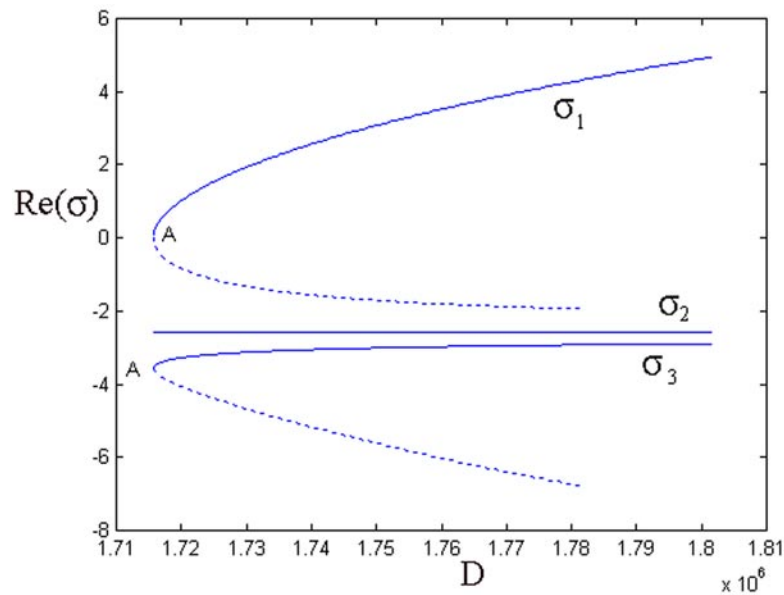


Figure 6. When $Le < 1$ the third eigenvalue diverges from the constant value obtained when $Le = 1$ by moving lower. As $Le \rightarrow 1$ σ_3 merges with σ_2 . $Pe = 0$, $Le = 0.95$, $T_a = 4$, $T_0 = 0.05$.

On the other hand, when Le is decreased *below unity*, σ_3 moves *under* the constant eigenvalue as shown in figure 6. The constant eigenvalue is still given by equation (8). Note that along the upper branch all the eigenvalues are negative, while along the middle branch σ_1

is positive and $\sigma_{2,3}$ are negative. Thus, for $Le < 1$ the upper branch is stable and the middle branch is unstable. The remaining constant eigenvalue disappears in the case $Le_O \neq Le_F \neq 1$.

It has been shown that the system behaviour changes when Le is increased passed unity. Damped oscillations appear when $1 < Le < Le_c$, but the entire upper branch remains stable. The beginning of the upper branch is unstable when $Le > Le_c$.

4.1. Cellular flames

'Cellular flames' have been observed in both freely propagating premixed flames [18], burner attached diffusion flames [19,20], and micro-gravity flame spread. All these systems are fairly complex, involving multi-dimensional processes that generally include flame-surface interactions. The qualitative nature of the cellular flame instability can be described by examination of the proposed one-dimensional problem.

In order to allow for spatial variation along a plane of unit depth, the $\partial^2/\partial x^2$ operator in equations (1a) and (1b) needs to be replaced with the Laplacian $\partial^2/\partial x^2 + \partial^2/\partial y^2$ and perturbations take the form $\phi'(x, y, t) = f(x) \exp(\sigma t + iKy)$ in place of equations (7a)–(7c). Here, K is a non-dimensional wavenumber in the new coordinate direction, y , which lies in the plane ((y, z) -plane, where z is the unit-depth coordinate) of the flame sheet. This coordinate dependence will show the development of undulations in the flame, whereas the one-dimensional formulation merely produced fluctuations about the mean flame position, x_f . When $Le = 1$, the new eigenvalues, σ_{new} , are simply equal to the old eigenvalues σ shifted by K^2 , i.e. $\sigma_{\text{new}} = \sigma - K^2$. When $Le \neq 1$ the relation between σ_{new} and σ is far more complex but qualitatively identical. In particular, the introduction of K has a stabilizing influence on the flame. Two conditions need to be met in order to see the flame stripes in the y -coordinate plane. First, the boundary control requires that K be a multiple of $2\pi/L$, where L is the dimensionless length (y -coordinate) of the burner. If, for example, the ratio between the length and the width of the burner is 10, then K can take values 0.31, 0.63, Second, the new leading eigenvalue has to be very close to zero to ensure the temporal changes of the flame structure do not rapidly vary and destroy the striped flame pattern. The first condition, pertaining more to experimental aspects, will be ignored in the rest of the discussion. For each unstable steady-state solution near point A on the 'S' curve a K value is computed, called K_{crit} , such that the new leading eigenvalue on the middle branch is identically zero. When $0 < Le < Le_c$ on the *middle* branch away from the upper turn on the 'S' curve, K_{crit} gradually increases from zero at point A as D is increased. When $Le > Le_c$, the leading eigenvalue is positive near the turn A; and, hence, a finite K , sufficiently greater than zero, is needed to stabilize the unstable steady solution, see figure 7. Therefore, it is clear from figure 7 that cellular flames will be more easily witnessed for conditions satisfying $Le < Le_c$.

Of particular interest is how thermal-diffusive effects contribute to the determination of cellular flame size. Additionally, the convection of fuel as described by Pe is examined to identify the hydrodynamic effects. Thermal-diffusive effects are examined by varying the Lewis number and holding Pe constant at 0.5. The results for this numerical experiment are illustrated in figure 7. It is apparent that the influences of varying Le are relatively small. The critical K values are only moderately changed between the cases $Le = 0.9$ and 0.3. This suggests that thermal-diffusive effects play only a minor role in determining the flame cell size. The distance, measured in D , away from the instability point A is the main determinant of the size of the flame cells

In order to determine the effects of fuel flow on the cellular flame size several different values of Pe were examined. The range of Pe was varied between 0 and 0.75, and Le was

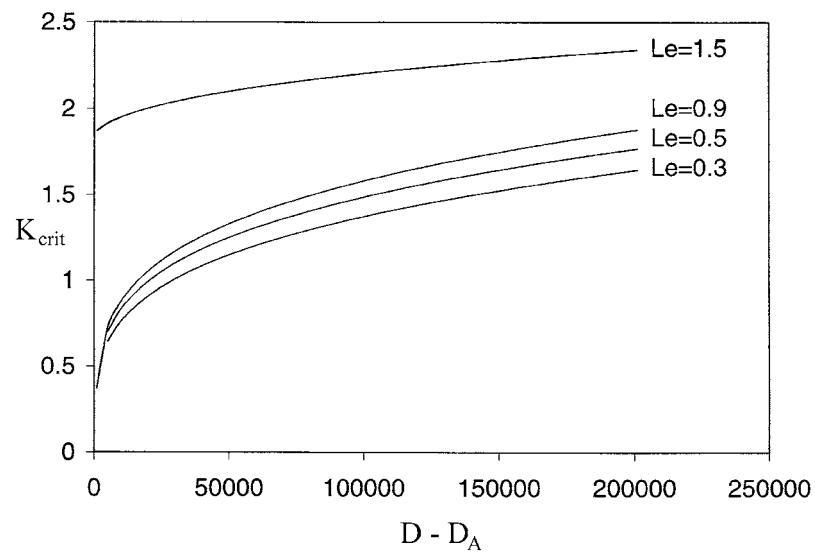


Figure 7. The thermal–diffusive properties of the reactants plays a significant role in determining the flamelet size as well as the location of the onset of instability. Here $Pe = 0.5$, $T_a = 5.0$, $T_0 = 0.05$.

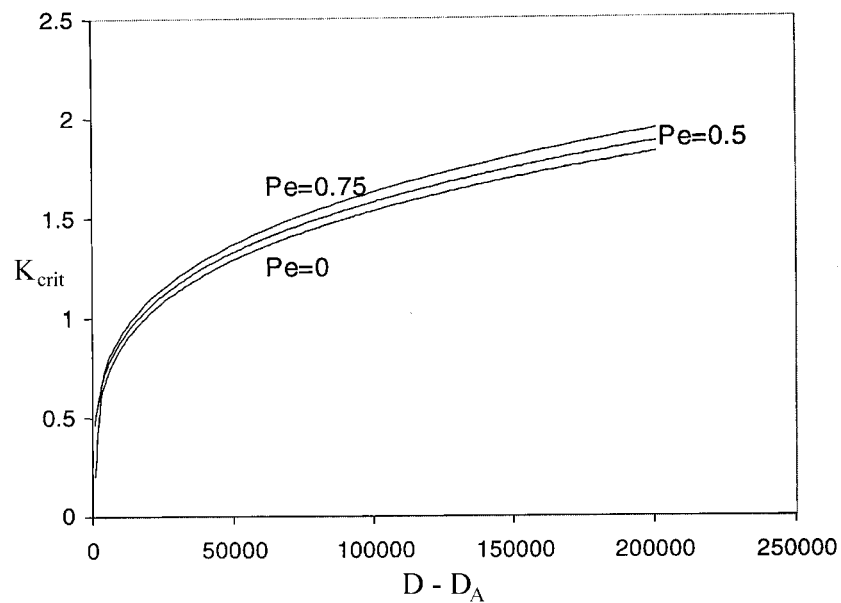


Figure 8. The critical wavenumber for $Le = 1$ for solutions along the unstable middle branch are relatively unaffected by the crossflow velocity. However, the increased cross flow results in a smaller flamelet after flame front breakup. $Le = 1$, $T_a = 5.0$, $T_0 = 0.05$.

set equal to unity so that the diffusion term and the convective term were approximately of the same order of magnitude. In this range of Pe , the ‘S’ curves retain their characteristics and are simply shifted towards higher D ranges. Figure 8 shows the K_{crit} values for the three different flow rates, $Pe = 0, 0.5$ and 0.75 . There is little difference between the K_{crit} values

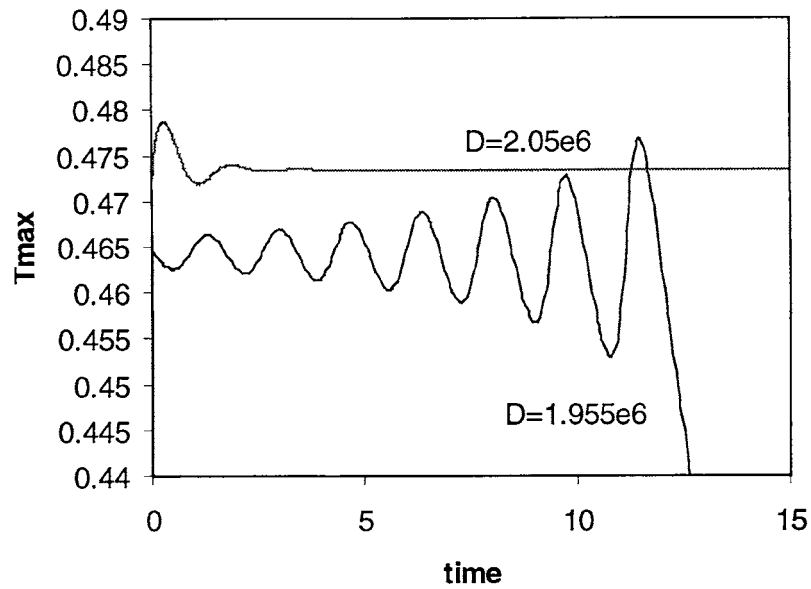


Figure 9. Transient solution for $Le = 2Pe = 0$, $T_a = 4$, $T_0 = 0.05$ at values of D on either side of point C' . In one case the oscillations grow with time and lead to instability. In the other, the oscillations decay leading to a stable solution.

for a zero convection diffusion flame ($Pe = 0$) and one for which convection is appreciable, i.e. $Pe = 0.75$. This leads to the conclusion that for order-unity fuel crossflow the size of the cellular flames is little affected.

4.2. Oscillatory behaviour ($Le > Le_c$)

Oscillatory behaviour in flames has been observed in micro-gravity candle experiments aboard the Mir space station, as well as during droplet burning experiments [21]. Several numerical investigations have examined oscillatory flames [4, 10]. The qualitative nature of this phenomenon can also be described with a linear stability analysis of a one-dimensional diffusion flame.

When $Le > Le_c$, the leading eigenvalues form a complex conjugate pair beyond the point B on the upper curve (see figures 2 and 5). The real part is positive at B and decreases as D increases. At point C' the real part reduces to zero although the imaginary part is non-zero. So, between point C' and C on the upper curve, small perturbations will result in damped oscillations. Between B and C' , small perturbations will oscillate with growing amplitude leading to flame extinction. A stable periodic orbit was not found numerically near point C' . Near C' , long-lasting oscillations are expected with circular frequency equal to the imaginary part of the leading eigenvalue pair.

The oscillatory behaviour predicted by the linear stability analysis was supported by solving equations (1a) and (1b) directly using a second-order correct finite-difference method. In doing so, a graphical representation of the transient nature of the flame was achieved as well as a search for possible steady oscillatory behaviour. The results in figure 9 represent transient solutions for two points along the upper branch. The unstable solution is representative of a D value below point C' ($D_{C'} = 1.961E + 6$), while the stable solution is for a D larger than that at point C' .

The numerical investigation of the oscillatory behaviour is similar to that performed by Kim *et al* [4]. Although the method is able to identify all the characteristics of this simplified problem that have been identified by the preceding stability analysis, it is time consuming and does not allow Le_c to be readily evaluated.

5. Discussion and conclusions

The mathematical examination of equations (1a) and (1b) and the linearized small perturbation equations (6a)–(6d) in terms of the first three eigenvalues leads to a clear picture of the overall phenomenon. The principal virtue of the model considered here is its simplicity and the absence of complications in it that might cause difficulties of interpretation. It has been stated that convection is ‘essential’ for instabilities to occur in the configuration of figure 1 [2]. Our examination produces results for $Pe = 0$ that are qualitatively identical with those for $Pe \neq 0$. It appears that a non-zero value of Pe does not alter the nature of the stability calculation, even though complications enter as Pe becomes large. The stability results for $Le < Le_c$ were qualitatively simple, as the results of section 4 demonstrate. The situation for $Le > Le_c$, however, was more complicated. Here, several possibilities existed, whose realization depended primarily on the movement of the locus of the first two eigenvalues as a function of D and Le . Since a complex conjugate pair is formed on the upper branch after point B, oscillatory behaviour there is the norm. Between A and B there are no oscillations. The critical value Le_c is the value of $Le > 1$ for which point B is located at the turning point A of figure 2.

It is possible that asymptotic analyses can provide approximate formulae for the information obtained here by numerical methods of eigenvalue computation. The asymptotic calculations, however, are tedious and lengthy (see, e.g., [2] for the $Le < 1$ case), and provide results in terms of parameters that are, in practice, difficult to calculate. An example of a difficult parameter to calculate is the Damköhler number derivative [2]. The calculation of the first few eigenvalues as outlined here, however, is in complete accord with the mathematical theory of systems of equations [16, 17]. The eigenvalue computation provides an immediate and complete assessment of the diffusion flame response, not only in the vicinity of specially chosen points of analysis (such as A) but everywhere along the entire ‘S’ curve.

The Nyquist-plot analysis of Kirkby and Schmitz [1] also provides mathematically rigorous results. Nevertheless, considerably greater effort is required to interpret and employ the results presented therein. The difference between this work and that of Kirkby and Schmitz [1] is not mathematical rigour but ease of interpretability and consistency with the function-theoretic analysis of partial differential equations.

For the reasons given in the previous two paragraphs, we believe that our examination provides a comprehensive and readily understood description of diffusion flame instability. Additional analysis of the eigenvalue spectrum is suggested by the present analysis for the case of large Pe (where the behaviour of the eigenvalues becomes more complicated) and when the values of Le differ for the two reactants. The permutations in the latter case are numerous, as there is a blowing reactant and a diffusing reactant; Le for both reactants may be small, large, one large the other small; and so on. In addition, the behaviour of the solution on the upper branch for $Le > Le_c$ near point B may warrant theoretical analysis because a bifurcation exists there (real parts of two complex conjugate eigenvalues cross from positive to negative values in the direction of increasing D).

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