Math Circle # Modular Arithmetic **Jan 21, 2013**

1a. Write the multiplication table for multiplication modulo 11 (on an 11-hour clock).

$\times$	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

Hints:

- Start by filling in the diagonals. For example $5 \times 5 = 25 \equiv 3 \pmod{11}$, since $25 = 2(11) + 3$.
- Count right from each diagonal square, counting by the number of the row. For example, from $4 \times 4 = 5$, fill in $4 \times 5 = 5 + 4 = 9$, then $4 \times 6 = 9 + 4 = 13 \equiv 2$, etc.
- Finally, fill in the columns below the diagonal from the corresponding rows by the commutative law: $5 \times 4 \equiv 9$, $6 \times 4 =\equiv 2$, etc.

b. Recall that the inverse a^{-1} means the mod-11 number which cancels a so that $a \times a^{-1} \equiv 1$. For example, $2^{-1} = 6$, since $2 \times 6 = 12 \equiv 1 \pmod{11}$. Find the inverse of every number $0, 1, 2, \ldots, 10$, if there is one.

2. Having the inverse a^{-1} allows us to divide by a, intepreting $b \div a$ as $b \times a^{-1}$. Find all solutions (if any) for x in the following equations:

a. $5x + 7 \equiv 3 \pmod{11}$

b. $x^2 \equiv 4$

c. $x^2 \equiv 3$

3a. If a whole number n is divisible by 11, then $n \equiv$ what number mod 11?

b. Try reducing mod 11 to see if 11 evenly divides this number: $243 = 2 \times 10^2 + 4 \times 10 + 3$.

c. Can you change one digit of 243 so that 11 does divide it?

d. Find a simple rule with the digits of a number (similar to the Rule of 3) to decide whether 11 divides evenly or not.