

For  $K$  and  $L$  two knots, let

$$M_{K,L,p} = (S^3 \setminus K) \cup_{l \sim l+pm} (S^3 \setminus L),$$

where  $l$  is the canonical longitude and  $m$  the meridian. We compute

$$QV_{2m+1}(M_{K,L,p}) = \frac{2\pi}{2m+1} \log |TV_{2m+1}(M_{K,L,p}, e^{\frac{2i\pi}{2m+1}})|$$

for each possible  $K$  and  $L$  among the figure eight knot, the  $5_2$  knot, the  $6_1$  knot and the  $(-2, 3, 7)$ -pretzel knot and  $|p| \leq 3$ . We have that

$$TV_{2m+1}(M_{K,L,p}, q) = \eta_m^4 \sum_{i=1}^m [i]^2 \left( (-1)^{i-1} q^{(i^2-1)/2} \right)^p J_i(K, q^2) \overline{J_i(L, q^2)}$$

where  $\eta_m = \frac{2 \sin(\frac{2\pi}{2m+1})}{\sqrt{2m+1}}$  and  $[n] = \frac{q^n - q^{-n}}{q - q^{-1}}$ .

The family of colored Jones polynomials values  $\{J_i(K, q^2)\}_{1 \leq i \leq m}$  of these knots are computed in linear time in  $m$  using the known  $q$ -difference relations for these knots. At  $m = 1000$ , we get the following table of values. In all cases,  $QV_{2001}(M_{K,L,p})$  comes within 0.4% of the actual volume.

$K$	$L$	$p$	$QV_{2001}(M_{L,K,p})$	$\text{Vol}(M_{L,K,p})$
$4_1$	$4_1$	-3	4.07384310	4.05976643
		-2	4.07398752	
		-1	4.07408105	
		0	4.07411352	
		1	4.07408105	
		2	4.07398752	
		3	4.07384310	
$5_2$	$5_2$	-3	4.87080285	4.85800530
		-2	4.87103827	
		-1	4.87125583	
		0	4.87144129	
		1	4.87157816	
		2	4.87165099	
		3	4.87165022	
$6_1$	$6_1$	-3	5.20554618	5.19384644
		-2	5.20579242	
		-1	5.20601218	
		0	5.20618659	
		1	5.20629554	
		2	5.20632364	
		3	5.20626653	

$K$	$L$	$p$	$QV_{2001}(M_{L,K,p})$	$\text{Vol}(M_{L,K,p})$
$4_1$	$(-2, 3, 7)$	-3	4.86654717	4.85800530
		-2	4.86669111	
		-1	4.86684156	
		0	4.86699909	
		1	4.86716435	
		2	4.86733804	
		3	4.86752097	
$5_2$	$5_2$	-3	5.66886952	5.65624418
		-2	5.66905660	
		-1	5.66918060	
		0	5.66922421	
		1	5.66918060	
		2	5.66905660	
		3	5.66886952	
$6_1$	$6_1$	-3	6.00364959	5.99208532
		-2	6.00381775	
		-1	6.00390081	
		0	6.00388436	
		1	6.00377145	
		2	6.00358087	
		3	6.00333822	
	$(-2, 3, 7)$	-3	5.66425507	5.65624418
		-2	5.66441833	
		-1	5.66459000	
		0	5.66477092	
		1	5.66496202	
		2	5.66516440	
		3	5.66537927	
$6_1$	$6_1$	-3	6.33814464	6.32792646
		-2	6.33838441	
		-1	6.33854811	
		0	6.33860671	
		1	6.33854811	
		2	6.33838441	
		3	6.33814464	
	$(-2, 3, 7)$	-3	5.99862065	5.99208532
		-2	5.99878023	
		-1	5.99894794	
		0	5.99912463	
		1	5.99931123	
		2	5.99950883	
		3	5.99971868	
$(-2, 3, 7)$	$(-2, 3, 7)$	-3	5.66790495	5.65624418
		-2	5.66850076	
		-1	5.66900980	
		0	5.66922421	
		1	5.66900980	
		2	5.66850076	
		3	5.66790495	