Probability Homework Solutions

1. To win the Ohio Super Lotto Plus lottery, you must correctly pick which six balls are chosen out of a collection of forty-nine numbered balls. What is the probability of winning the Ohio Lottery if you have one ticket?

   Answer: The number of possible outcomes (each equally likely) is ${}_{49}C_6 = 13983816$, but only one outcome is a win for you. Therefore, the probability of winning is 1/13983816

2. In the NBA lottery, 14 numbered ping pong balls are placed in an urn, and four are drawn out at random (unordered) to determine who gets the first pick in the draft. The team with the worst record is given 250 numerical combinations, the team with the second worst record is given 200, the team with the next worst record 157, and so on, with the team with the 13th worst record getting 1 combination. (All other teams make the playoffs and are not eligible for the lottery.) What is the probability that the worst team gets the first pick? What about the 3rd worst team? What about the best of the eligible teams?

   Answer: There are ${}_{14}C_4 = 1001$ possible combinations all together and the worst team has 250 of these combinations, so the worst team has probability 250/1001 of receiving the top pick. The team with the third worst record has 157 balls, and a probability of 157/1001. The best of the eligible teams has one combination, and hence a probability of 1/1001

3. In Las Vegas, one of the popular games is called "birdcage." The game works by rolling three standard six-sided dice, and you win if at least one die comes up 6. What are the odds of your winning?

   Answer: There are $6 \times 6 \times 6 = 216$ ways for the dice to come up, and $5 \times 5 \times 5 = 125$ of these ways have no sixes coming up. (We counted E'.) Thus the odds of winning are $86 : 125$ since $86 + 125 = 216$.

4. Suppose everyone in the class chose a "secret" 3-digit PIN number. What is the probability that everyone chooses a different number? Assume there are 120 students in the class.

   Answer: The number of 3-digit PIN numbers is 1000 (000-999). The number of ways that the class could have chosen all different PIN numbers is ${}_1000P_{120}$, while the number of ways the class could have chosen all PIN numbers is 1000^{120}. At this point we need to use a calculator or spreadsheet to calculate as we did in the birthday problem. In particular, the probability of the first two PIN numbers being different is $1 - (1/1000)$. The probability that three people pick different PIN numbers is

   $$(1 - 1/1000) \times (1 - 2/1000).$$

   The probability for 120 people is then

   $$(1 - 1/1000) \times (1 - 2/1000) \times \cdots \times (1 - 119/1000),$$

   or approximately .000586.

   Curiously, even if everyone were to choose a four-digit PIN, the probability of two students having chosen the same PIN numbers is still over 50%.

5. A family has 6 children. Assume that each child is equally likely to be a boy or a girl. The children are listed in the order of birth, e.g., (b,b,g,g,b,g)

   a. How many such listings are possible?

   b. What is the probability that the family has exactly 3 girls. Evaluate your answer as a decimal.

   c. What is the probability the family has at least 4 boys?

   d. What is the probability of at most 2 girls?

   Answer: a. 64, b. ${}_6C_3/2^6 = 5/16 = 0.3125$, c. (1+6+15)/64 , d. same as c.

   For d., there is one way to have no girls, there are six ways to have one girl and five boys, and there are ${}_6C_2 = 15$ ways to have two girls and four boys, so the total number of ways of having 0, 1, or two girls is $1 + 6 + 15 = 22$.

6. A roulette wheel has both a 0 and 00 and the numbers from 1 to 36. When you spin the wheel, the ball will land on one of the numbers from 1 to 36, or else on 0 or 00. You get to choose two numbers from the 38 "numbers" $\{0, 00, 1, 2, 3, \ldots 36\}$. The payoff is 17: 1 (You receive back your stake plus 17 times your stake.) What is the probability of winning? What are the odds?

   Answer: Suppose you have placed your bet, so you have chosen two of the 38 possible numbers. When the ball lands, there are 2 ways it can land so you have a success. There are 38 ways it can land in all. So $n(E)/n(S) = 2/38 = 1/19$, and the odds are $2\colon 36 = 1\colon 18$. Notice that the payoff has nothing to do with the probability of winning. The payoff has to do with the expected value, which will be discussed soon.

7. The odds against your winning a game are 7: 2. What is the probability you will lose?

   Answer: The odds against your winning a game are the odds that you will lose when you play the game. So the answer is 7/9.

8. Although all 12 of the doughnuts on the tray in the cafeteria look delicious, four of the doughnuts were left over from yesterday and are stale. You randomly take three doughnuts. Find the following probabilities:

   (a) All 3 are stale.

   Answer:

$$\frac{{}_4C_3}{{}_{12}C_3} = \frac{1}{55} = 0.0181818$$

   (b) All three are fresh.

   Answer:

$$\frac{{}_8C_3}{{}_{12}C_3} = \frac{14}{55} = 0.254545$$

(c) Exactly one is stale.

Answer:

$$\frac{{}_4C_1 \times {}_8C_2}{{}_{12}C_3} = \frac{28}{55} = 0.509091$$

(d) At most one is stale.

Answer: 3 fresh or 2 fresh:

$$\frac{{}_8C_3 + {}_8C_2 \times {}_4C_1}{{}_{12}C_3} = \frac{42}{55} = 0.763636$$

(e) At most one is fresh.

Answer: all stale or 2 stale:

$$\frac{{}_4C_3 + {}_4C_2 \times {}_8C_1}{{}_{12}C_3} = \frac{13}{55} = 0.236364$$

9. On a particular day 80 students attended a math 106 lecture. Make the usual assumption that every year has 365 days, so we avoid the complication of leap years. What is the probability that at least two people who attended the lecture have the same birthday?

Answer:

n(S)=the number of ways 80 people can have birthdays = 365^{80}. Let n(E)=the number of ways thirty people can have different birthdays. $n(E) = {}_{365}P_{80}$. So we have

$$p(E) = \frac{n(E)}{n(S)} = \frac{{}_{365}P_{80}}{(365)^{80}}$$

If you use an HP38G calculator, you can get the answer. You enter $\text{PERM}(365, 80)/365 \wedge 80$. If you use a TI calculator, you would push 365, MATH, PRB, then nPr, then 80, ENTER, and then divide by 365^{80}. However, if you try that, you will get an error message due to overflow (The numerator is too large for the calculator.) Instead, open an Excel worksheet. In a cell you enter

$= \text{PERMUT}(365, 80)/(365 \wedge 80)$. The equals sign at the beginning of the entry means you are entering a formula. When you enter the formula and push the return key, you will see that the answer is 8.57 E-5. This means that the probability of 80 people all having different birthdays is approximately 8.57×10^{-5}. The probability that at least two people have the same birthday is then

$$1 - 8.57 \times 10^{-5} \approx .999914332$$

In order to try to understand this answer, you can check that

$$\frac{1}{2^{14}} \approx 6.1 \times 10^{-5}.$$

We note that 6.1×10^{-5} is reasonably close to 8.57×10^{-5}.

So the probability that at least two of 80 random people all having different birthdays is approximately the same as the probability that if a fair coin is tossed 14 times, that it will land tails all 14 times.

To put it another way, the probability that at least two of 80 random people have the same birthday is approximately the same as the probability that if a fair coin is tossed 14 times, that it will land heads at least once.