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Guts of surfaces and the colored Jones polynomial. (English) Zbl 1270.57002

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This monograph puts forward and supports the idea that the Euler characteristic of a certain graph associated to a link (a ‘reduced state graph’ of a ‘semi-adequate link’) provides a good complexity measure for the geometry and topology of the link complement. Its main results, whose proofs are combinatorial and geometric topological, are suggested by the way in which the abovementioned Euler characteristic arises from coefficients of colored Jones polynomials (quantum topological link invariants) taken together with conjectures that relate quantum and geometric invariants, chief among these being the Slope Conjecture and the Volume Conjecture.

In a certain sense, the results in this monograph bridge a gap between quantum and geometric topology. In geometric topology, a link is a finite collection of disjoint smooth embeddings of circles in the 3-sphere. A link is studied via its complement, that is a certain sort of 3-manifold. Given a spanning surface for a link, the interesting part of the surface complement is the portion of its JSJ decomposition that admits a hyperbolic metric with totally geodesic boundary, known as the “guts” of the surface. Conversely, in quantum topology, a link is a link diagram, i.e. a planar 4-valent graph with over-under information at vertices, modulo Reidemeister moves. To a link diagram and to a choice of ‘state’ (A or B) for each crossing, there corresponds a certain graph called the *state graph*. By previous results of Dasbach, Lin, Stoltzfus, and the first two authors, the coloured Jones polynomial can be computed from the state graph G_A obtained from choosing all A resolutions, or alternatively from the state graph G_B obtained from choosing all B resolutions. Removing duplicate edges gives rise to the *reduced state graph*. If either G_A or G_B contains no single-edge loops (in this case the link diagram is said to have been *semi-adequate*), then Dasbach and Lin have shown that the Euler characteristic of the relevant state graph arises from the second-highest (or the second-lowest) coefficient of the coloured Jones polynomial.

Thus, the monograph relates the reduced state graph, which is a combinatorial object whose study is motivated by quantum topology, with guts of essential spanning surfaces of the link, which belong to geometric topology. In particular, its results suggest that the ‘penultimate’ coefficients of the coloured Jones polynomial measure the distance of essential spanning surfaces in the link complement from being a fibre (if the relevant coefficient vanishes, the spanning surface is a fibre, while if it is large, then the surface is far from being a fibre).

In Chapter 9, motivated by the Volume Conjecture, new tighter bounds on the hyperbolic volume of link complements are obtained, for certain classes of links, in terms of the Euler characteristic of the reduced state graph. Unlike previous such bounds, the bounds in this monograph have the advantage that there are intrinsic and conceptual explanations for why they exist. Thus, the results provide conceptual support for the Volume Conjecture, and more generally for the idea that asymptotics of coloured Jones polynomials are related to geometric invariants of link complements. Indeed, they suggest a new such conjecture, the Coarse Volume Conjecture of Chapter 10.3.

The technical core of the monograph is normal surface theory inside a polyhedral decomposition of a surface complement. In Chapter 10.2, it is pointed out that these tools offer an avenue of attack on several conjectures including the Cabling Conjecture and the determination of hyperbolic A-adequate knots.

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57-02 Research monographs (manifolds)

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