Volume and it's relation to knot homology theories

Dear Effie,

Here is some data on 5,365 hyperbolic 1-1 knots with at most 16 crossings. It may not be a complete list of such, but it is probably pretty close.

```
In[1]:= NotCommentLine[L_] := Characters[L[[1]]][[1]] ≠ "#";
        data = Select[
           Import[ "/Users/dunfield/work/effie/one-one-data", "TSV"],
           NotCommentLine];
```

Functions for accessing the data.

```
In[3]:= Volume[L_] := L[[2]];
         HFK[L_] := L[[3]];
         Kh[L_] := L[[4]] ;  SecondCoeffs[L_] := L[[5]] ;
        AllCoeffs[L_] := ToExpression[StringJoin[ "{", StringTake[L[[6]], {2, -2}], "}"]];
        SumCoeffs[L_] := Plus @@ Map[Abs, AllCoeffs[L]];
```

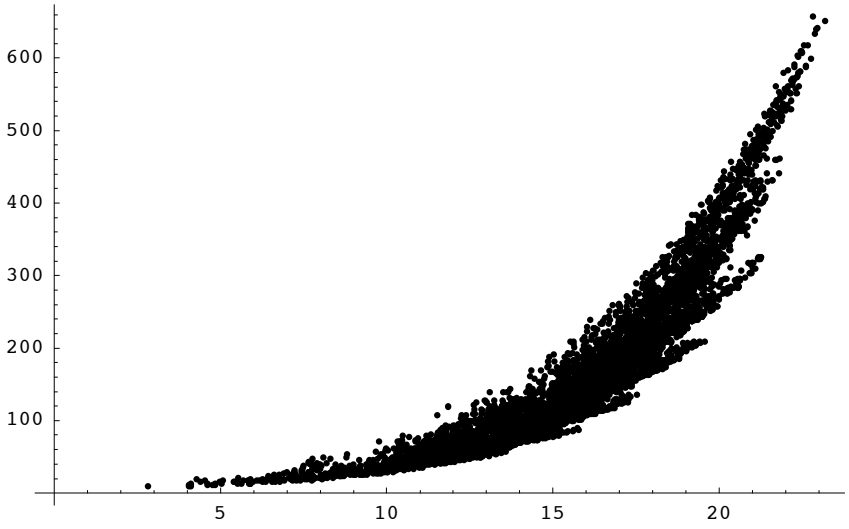
The ranks of HFK and the (reduced) Khovanov homology are nearly always the same, except in two examples. Even there the difference is very minor.

```
In[8]:= Select[ data, HFK[#] ≠ Kh[#] &] // TableForm
```

```
Out[8]//TableForm=
       16n207543     4.60993     13     15     2     [-1, 1, -1, 1, -1, 1, -1]
       16n246032     5.4954      15     17     1     [1, -1, 1, -1, 1, -1, 0, 0, 0, 1]
```

Here is a plot of volume vs. rank of HFK/Kh

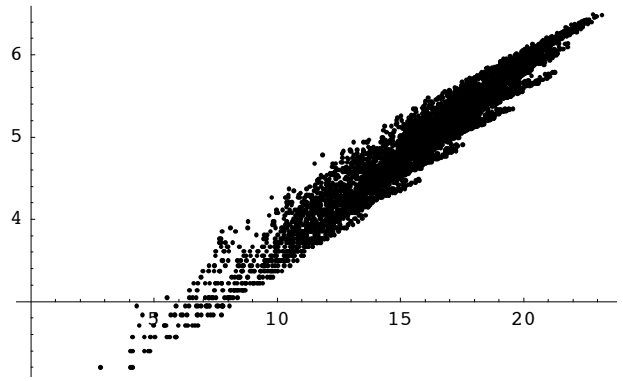
```
In[9]:= ListPlot[ Map[ {Volume[#], HFK[#]} &, data]]
```




```
Out[9]= - Graphics -
```

The picture improves (as expected) if we take the log of the rank of the homology.

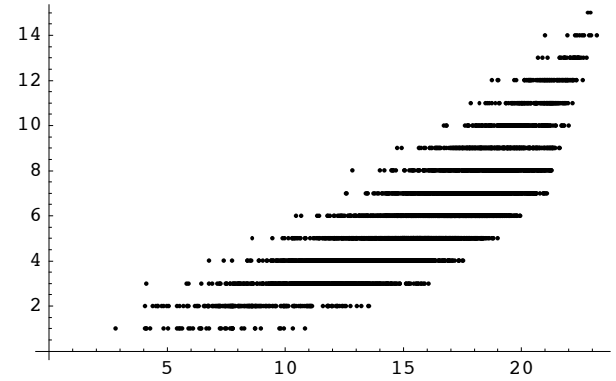
```
In[10]:= ListPlot[ Map[ {Volume[#], Log[HFK[#]]} &, data]]
```




```
Out[10]= - Graphics -
```

Now, let's compare the volume to the sum of the absolute value of the "almost leading" terms of the Jones polynomial :

```
In[11]:= ListPlot[ Map[ {Volume[#], SecondCoeffs[#]} &, data]]
```




```
Out[11]= - Graphics -
```

Thus clearly there are approximately linear relationships between volume and both Log[Kh] and SecondCoeffs. I will now investigate this further to see whether there are possible inequalities between these quantities, as there is for alternating knots in the case of SecondCoeffs. To do this, we simply consider the ratios of Volume to these two quantities.

```
In[12]:= KhData := Sort[Map[ Volume[#] / Log[Kh[#]] &, data]];
         JonesData := Sort[Map[ Volume[#] / SecondCoeffs[#] &, data]];

         Map[ {Min[#], Mean[#], Max[#]} &, {KhData, JonesData}] // TableForm
```

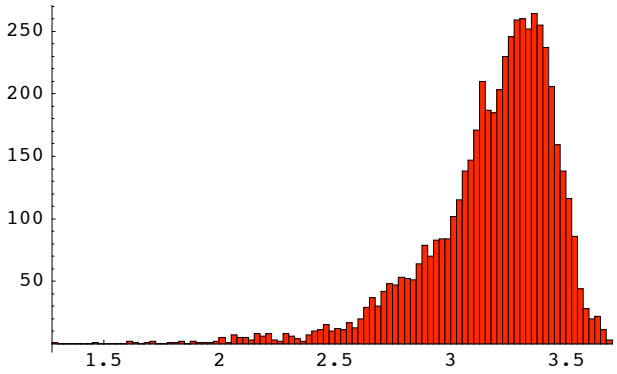
```
Out[14]//TableForm=
       1.28713      3.18151      3.67829
       1.37497      2.94682      10.8461
```

```
In[15]:= << Graphics`Graphics`
```

Here are histograms for these two data sets. They seem to suggest that only one-sided bounds on these quantities can be expected. Very conveniently, these bounds are from the opposite sides.

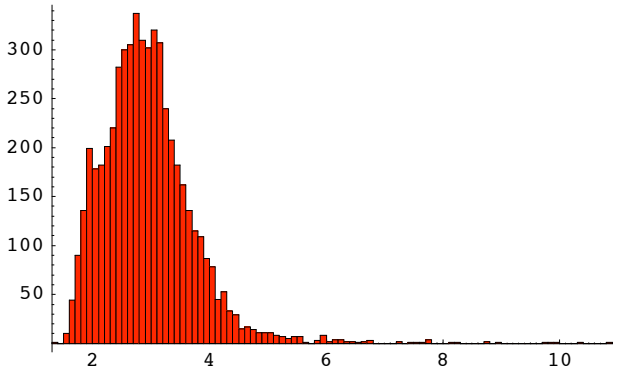
If I'm thinking correctly, there should be no lower bound for Volume/Kh even for 2-bridge knots (essentially because the same thing fails for Log(Jones(-1))).

In[16]:= **Histogram[KhData]**



Out[16]= - Graphics -

In[17]:= **Histogram[JonesData]**



Out[17]= - Graphics -