Math 996: Topics in Knot THeory
Fall 2015

Lecture: MTH 996

Location: A228 WH
Schedule: days: MWF 11:30 AM - 12:20 PM

Instructor:

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Overview/Scope/ Prerequisites: The overarching goal of the course is to give a broad introduction to knot theory and the many areas of low-dimensional topology with which it interacts. It will take participants from the beginnings of the subject to the forefront of exciting recent developments. Emphasize will be placed on open problems, conjectures and current directions of research, and the course should provide stimulation for further independent reading and research. The only formal prerequisite will be to have taken 868-869 or the equivalent.

Description: Topics will be selected from: Knot diagrams and their in- variants, Braid groups, Seifert Forms, Cyclic covers of knot complements and their invariants, Jones type polynomial invariants, quantum invariants, surfaces in knot complements, hyperbolic knots, relations between quantum invariants and geometric structures of knots and concordance groups.

Some textbooks:

- Cromwell: *Knots and links* by Peter Cromwell
- Lickorish: *An introduction to knot theory* by W.B.Raymond Lickorish.
- Murasugi: *Knot theory* by Kunio Murasugi
- Prasolov-Sossinsky: *Knots, links, braids and 3-manifolds* by V.V.Prasolov and A.B.Sossinsky
- Rolfsen: *Knots and links* by Dale Rolfsen

Outline and References:

Note: The text in Red shows hyperlinks. Click for more details.

1. Knots and links : Topics selected from: Definitions, diagrams, Reidemeister moves, wild knots, connect sum, unknotting, linking number, Dowker-Thistlethwaite code, satellite knots, famous families of knots, **historical overview of knot theory.**

   Reading: Lickorish ch 1, Cromwell ch 1-4, Prasolov-Sossinsky ch 1.

2. Braids and tangles: Topics selected from: Braid group, Alexander and Markov theorems, braid index. Rational tangles, rational links, 2-bridge links, plats.

   Reading: Prasolov-Sossinsky ch 3, Cromwell ch 8, 10.1, 10.4.

3. Seifert surfaces and Seifert matrices: Topics selected from: Seifert surfaces, circuits and genus, Yamada's theorem and Vogel's algorithm. Seifert matrix, S-equivalence. Signature and determinant.

   Reading: Lickorish ch 2,8, Cromwell ch 5-6, Murasugi ch 5, 6.4.

4. Alexander polynomial: Topics selected from: Many approaches to Alexander polynomial: Seifert matrices, homology of infinite cyclic cover, Fox calculus, Conway polynomial. Wirtinger presentation for the knot group.

   Reading: Lickorish ch 6,7,11, Cromwell ch 7, Rolfsen ch 6-8, Murasugi ch 6.

5. Jones polynomial I: Topics selected from: Kauffman bracket, state sums. Alternating knots and proof of Tait's conjecture. Adequate diagrams. Turaev surface and genus.
   Reading: Lickorish ch 3,5, Prasolov-Sossinsky ch 2.3, Cromwell ch 9.

6. Jones polynomial I: Topics selected from: Tait graph and spanning-tree expansion for the Jones polynomial. Tutte polynomial of graphs. Temperley-Lieb algebra and braid representations.

   Reading:

   - Bollobas Modern Graph Theory ch 10,
   - **Jones Expo 1**,
   - **Jones Expo 2**,
   - **The Jones polynomial and graphs on surfaces** .

7. Quantum invariants: Topics selected from: Jones-Wenzl idempotents, colored Jones polynomial, Representations, R-matrices and Yang-Baxter equation, colored Jones polynomial, cabling formula.

   Reading: Lickorish ch 13-14, Prasolov-Sossinsky ch 8, **Masbaum-Vogel** , Ohtsuki Quantum Invariants ch 4.

8. Hyperbolic knots: Topics selected from: Hyperbolic geometry of 3-manifolds, figure-8 knot complement, ideal tetrahedra, gluing and completeness equations.

   Reading:

   - **Purcell**,
   - **Adams**,
   - **Weeks**.

9. Hyperbolic volume: Topics selected from: Hyperbolic volume, properties, diagrammatic bounds.

   Reading:

   - Ratcliffe: Foundations of hyperbolic manifolds chapter 10,
   - **Milnor**,

- **Lackenby-Agol-DThurston**,
- **Futer-Kalfagianni-Purcell**,
- **SnaPy**,
- **Guts of surfaces and the colored Jones polynomial**, chapter 9.

10. Relations of Quantum invariants to geometry: Open Questions and Progress.

    Reading:

    - H. Murakami : **An Introduction to the Volume Conjecture**,
    - **Guts of surfaces and the colored Jones polynomial**,
    - **Jones polynomials, volume, and essential knot surfaces: A survey**

Grade: Your grade in this course will be determined by attendance, participation in class discussions and questions, and a presentation or a written "take home" assignment.